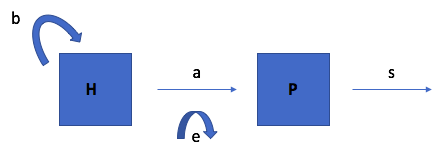
Final Project - Poterek/Portmann

Note: Both models are in the file finalProject.py, while the files LV.py and RM.py show our commits and changes.

# Lotka-Volterra

## Conceptual Model



## Questions

**The role of birth rate, “b”**

* The prey birth rate or “b” helps determine the maximum population of prey and predator.
* Increasing the prey birth rate increased the population of the herbivore as well as the population of the predator by the same proportion. The population of predators increased as the prey population increased because there is more food available.
* Decreasing the prey birth rate decreased the maximum population of the herbivore and the max population of the predator by the same proportions.
* Increasing the prey birth rate shortened the cycle length.
* Decreasing the prey birth rate lengthened the cycle length.

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| Figure 1: Lotka-Volterra Model with suggested parameters, b=0.5 | Figure 2: Lotka-Volterra Model with an increased herbivore birth rate, b=1.0 |
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| Figure 1 (repeated): Lotka-Volterra Model with suggested parameters, b=0.5 | Figure 3: Lotka-Volterra Model with a decreased herbivore birth rate, b=0.2 |

**The role of predator Attack Rate, “a”**

* The predator attack rate seemed to play a role in determining the proportion of predator to prey. If the predator attack rate was higher, the population sizes were more comparable, although the herbivore population still exceeded the predator’s. If the predator attack rate was low, the herbivore population way exceeded the predator’s population. The difference between the populations decreased as the predator attack rate increased because predators were getting more food and had a higher fitness, and the prey was getting eaten more.
* Increasing the predator attack rate decreased the maximum prey and predator populations.
* Increasing the predator attack rate shortened the length of a cycle, allowing for more cycles in the set time frame. This is the case because the turnover of prey is faster because of the higher attack rate allowing for faster cycles.
* Decreasing the predator attack rate "a" increased the maximum prey population, therefore increasing the maximum predator population.
* Decreasing the predator attack rate lengthened the cycle time, so there were fewer cycles in the set time frame.

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| Figure 1 (repeated): Lotka-Volterra Model with suggested parameters, a = 0.02 | Figure 4: Lotka-Volterra Model with increased predator attack rate, a = 0.04 |
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| Figure 1 (repeated): Lotka-Volterra Model with suggested parameters, a = 0.02 | Figure 5: Lotka-Volterra Model with decreased predator attack rate, a = 0.01 |

**The role of conversion efficiency of prey to predators, “e”**

* The conversion efficiency role was to control the population size of the prey.
* Increasing the conversion efficiency did not affect the maximum predator population, but it lowered the maximum prey population. This is because with more energy per meal, the predator population will rise and the prey population will fall. The lack of prey will then combat the rise in number of predators, leaving the maximum predator population constant.
* Decreasing the conversion efficiency also did not affect the max predator population, but it raised the maximum prey population.
* Changing the conversion efficiency did not affect cycle length.

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| Figure 1 (repeated): Lotka-Volterra Model with suggested parameters, e = 0.1 | Figure 6: Lotka-Volterra Model with an increased prey to predator conversion efficiency, e = 0.2 |
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| Figure 1 (repeated): Lotka-Volterra Model with suggested parameters, e = 0.1 | Figure 7: Lotka-Volterra Model with a decreased prey to predator conversion efficiency, e = 0.05 |

**The role of predator death rate, “s”**

* The predator death rate’s role was to control the size of the prey population.
* Increasing the predator death rate did not affect the maximum predator population, which seemed to stay around 100, but it increased the maximum prey population
* Increasing the predator death rate also shortened the length of a cycle.
* Decreasing the predator death rate again did not affect the maximum predator population, but lowered the maximum prey population.
* Additionally, decreasing the predator death rate increased the length of one cycle. This was the case because decreased death rate allows the prey population stabilize longer at a higher population size.

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| Figure 1 (repeated): Lotka-Volterra Model with suggested parameters, s = 0.2 | Figure 8: Lotka-Volterra Model with an increased predator death rate, s = 0.4 |
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| Figure 1 (repeated): Lotka-Volterra Model with suggested parameters, s = 0.2 | Figure 9: Lotka-Volterra Model with a decreased predator death rate, s = 0.1 |

**Summary:**

Parameters that affected only herbivores directly, such as herbivore birth rate, controlled the maximum population size of prey *and* predator. The increased populations of both predator and prey when the herbivore birth rate increased led to a shorter cycle length and vice versa.

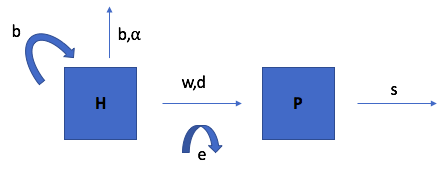
Parameters that affected only the predators directly, such as predator death rate, did not decrease the maximum predator population but affected the maximum herbivore population. A smaller predator death rate allowed for longer cycles because there was less turn over in the population.

Parameters that worked between predator and prey, such as predator attack rate and conversion efficiency, had varying results on the population dynamics of herbivore and predator. Increasing the predator attack rate raised the predator population and lowered the herbivore population, and shortened the cycle length. Decreasing the predator attack rate lowered the predator population and raised the herbivore population, and lengthened the cycle time. The conversion efficiency controlled the herbivore population, but had no effect on the maximum predator population. Increasing the conversion efficiency lowered the maximum prey population, and decreasing the conversion efficiency raised the maximum prey population.

The Lotka-Volterra consumer-resource model used the parameters herbivore birth rate, predator attack rate, conversion efficiency, and predator death rate to demonstrate the predator and prey relationship.

# Rosenzweig-MacArthur

## Conceptual Model



## Questions

**The role of birth rate, “b”**

* Increasing the prey birth rate "b" did not change the final prey population, likely because of the system's carrying capacity (or prey self-limitation), but it did increase the prey’s population initially and hastened approach to its final equilibrium. This initial increase in prey population density is likely responsible for the slight increase in final predator population, which also achieves equilibrium as the prey population remains constant.
* Decreasing the prey birth rate "b" decreased the final equilibrium population of both the prey and predator; as the prey population decreased due to a smaller influx of new individuals, the predators had less available food and thus could not sustain a large population
* In contrast with the dynamics described above, increasing “b” in the Lotka-Volterra (LV) model caused an increase in the prey population (as the LV model features no prey self-limitation). Additionally, manipulating “b” in the Rosenzweig-MacArthur (RM) model did not induce the cyclical behavior seen in the LV model, but rather an initial spike in each population followed by an eventual equilibrium.

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| Figure 1: Rosenzweig-MacArthur Model with suggested parameters | Figure 2: Rosenzweig-MacArthur Model with an increased herbivore birth rate, b=1.0 |
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| Figure 1 (repeated): Rosenzweig-MacArthur Model with suggested parameters | Figure 3: Rosenzweig-MacArthur Model with a decreased herbivore birth rate, b=0.3 |

**The role of predator Attack Rate, “a”**

* Decreasing the predator attack rate "a" caused oscillations in both populations; the prey population oscillated between a number much higher than in the original case (as a lower attack rate means fewer prey killed with each iteration) which was likely in excess of the system's carrying capacity. This caused the prey population to drop to ~0 immediately after reaching this peak due to the self-limitation parameter. The predator population also experienced much smaller oscillations that paralleled those occurring in the prey.
* Increasing the predator attack rate "a" decreased the final prey and predator populations (which both found an equilibrium), as more prey were consumed, thus decreasing the population over time and by extension decreasing the amount of food available for the predators.
* An increased “a” induced the same general decrease in both populations in the RM model as it did in the LV model. A decreased “a” caused the behavior of the RM model to mirror the cycles visible in the LV model.

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| Figure 1 (repeated): Rosenzweig-MacArthur Model with suggested parameters | Figure 4: Rosenzweig-MacArthur Model with increased predator attack rate, a = 0.002 |
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| Figure 1 (repeated): Rosenzweig-MacArthur Model with suggested parameters | Figure 5: Rosenzweig-MacArthur Model with decreased predator attack rate, a = 0.0005 |

**The role of conversion efficiency of prey to predators, “e”**

* Increasing the conversion efficiency caused oscillations in both populations, as the populations moved from low-prey and high-predator (as “e” caused more prey to be converted to predators) to high-prey and low-predator (as the predators ran out of food and decreased in number, allowing the prey population to rebound) and back.
* Decreasing the conversion efficiency increased the final prey population and decreased the final predator population, as the predators were less able to kill prey and thus declined in number, while the prey population no longer faced heavy predation and was only limited in number by the self-limitation parameter/carrying capacity.
* Just as in the LV model, increasing the conversion efficiency in the RM model did not affect the maximum predator population; it also induced oscillations like those in LV. Decreasing the conversion efficiency in LV and RM raised the maximum prey population, but the presence of the self-limitation factor in RM prevented any cyclical or exponential behavior in this model.

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| Figure 1 (repeated): Rosenzweig-MacArthur Model with suggested parameters | Figure 6: Rosenzweig-MacArthur Model with an increased prey to predator conversion efficiency, e = 0.1 |
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| Figure 1 (repeated): Rosenzweig-MacArthur Model with suggested parameters | Figure 7: Rosenzweig-MacArthur Model with a decreased prey to predator conversion efficiency, e = 0.04 |

**The role of predator death rate, “s”**

* Decreasing the predator death rate caused oscillations in both populations; as fewer dying predators caused the predator population to increase, the prey population was decimated as a result. This lack of food caused the predator population to drop and thus permitted the prey population to recover. The peak of the oscillations in both populations was above previous final values and the low point in both populations were close to zero.
* Increasing the predator death rate increased the final prey population and decreased the final predator population (as they attained equilibrium). As fewer predators were present to kill the prey, the prey population was essentially limited only by the carrying capacity of the environment.
* While both the LV and RM models’ manipulation of “s” caused changes in the prey populations, the RM model’s predator population also reflected changes in “s”. Generally, the primary difference between the LV and RM models here was because of its two additional parameters, which imposed limits upon both prey and predator maximum populations. In the case of Fig. 8, this means that the prey cannot exceed its final equilibrium value because the environment cannot support it. In Fig. 9, this means that only so much prey can be converted (and thus the predator population can only grow so large), in addition to the prey population self-limitation.

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| Figure 1 (repeated): Rosenzweig-MacArthur Model with suggested parameters | Figure 8: Rosenzweig-MacArthur with an increased predator death rate, s = 0.4 |
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| Figure 1 (repeated): Rosenzweig-MacArthur Model with suggested parameters | Figure 9: Rosenzweig-MacArthur Model with a decreased predator death rate, s = 0.1 |

**The role of the prey self-limiting factor, “w”**

* Increasing the self limiting factor caused oscillations in both populations; the peak of the oscillations in both populations was slightly below the final equilibrium value in that population with the original parameters (which reflects how fewer prey can be sustained by the environment), while the low point in both populations were close to zero. This cyclical pattern reflects the fluctuations between higher-prey and lower-predator situations (since prey populations can increase when there are fewer predators) and higher-predator and lower-prey situations (as predators respond to the higher-prey situation and increase in population, while the increase in predators causes prey populations to then drop).
* Decreasing the self limiting factor increased the final prey population and decreased the final predator population, as the prey population was less limited and thus able to grow more, while the predator population experienced less inflow as “w” was decreased and thus the population dropped.
* This parameter does not feature in the LV model, but distinguishes the RM model from LV as it contributes to the final equilibrium populations seen in RM (rather than the cycles visible in LV).

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| Figure 1: Rosenzweig-MacArthur Model with suggested parameters | Figure 10: Rosenzweig-MacArthur Model with an increased prey self-limitation rate, w=7.0 |
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| Figure 1 (repeated): Rosenzweig-MacArthur Model with suggested parameters | Figure 11: Rosenzweig-MacArthur Model with a decreased prey self-limitation rate, w=2.0 |

**The role of predator saturation response rate, “d”**

* Increasing the predator saturation response rate increased the final prey population and decreased the final predator population. The more saturated the predator population, the fewer prey could be consumed (and thus the fewer converted to predators). This increase meant less outflow from the prey population and less inflow into the predator population.
* Decreasing the predator saturation response rate caused oscillations in both populations; the peak of the oscillations in the prey population was above the previous final value and the low point in both populations were close to zero. This reflects the high rate of outflow from the prey population and the corresponding inflow into the predator population, as well as the shifts between high-prey low-predator and low-prey high-predator states (with dynamics as described previously).
* This parameter does not feature in the LV model, but distinguishes the RM model from LV as it contributes to the final equilibrium populations seen in RM (rather than the cycles visible in LV).

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| Figure 1: Rosenzweig-MacArthur Model with suggested parameters | Figure 12: Rosenzweig-MacArthur Model with an increased predator saturation rate, d=700 |
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| Figure 1 (repeated): Rosenzweig-MacArthur Model with suggested parameters | Figure 13: Rosenzweig-MacArthur Model with a decreased predator saturation rate, d=200 |

**Summary:**

* The primary difference between the RM model and the LV model is the presence of “w”, the prey self-limiting factor, and “d”, the predator saturation response. These parameters are responsible for the final equilibrium values seen in some situations (particularly when the original parameters were applied here), rather than the consistently cyclical behavior seen in the .LV model, as they impose limits upon prey and predator populations.
* In the LV model, cycle amplitude depends on initial conditions and results are not always realistic or reflective of actual population behaviors. It is assumed that the prey population grows exponentially indefinitely in the absence of predators, and the rate at which predators is unrealistic as well, as it continues to increase and essentially implies that an individual predator can never be full.
* The aforementioned parameters address these behaviors and impose limits upon modeled systems that better reflect real-world population behaviors, which are always subject to the limits of the environment and its biological capacity.

# Paradox of Enrichment

**What happens as the carrying capacity increases?**

As the carrying capacity of the prey increases, the stability of the predator and prey populations decrease. The instability in both the predator and prey populations can be visualized on the graphs below by the cycles of ups and downs. These cycles are present when the carrying capacity is high at 2000 (α = 0.0005), but not present when the carrying capacity is lower at 800 (α = 0.00125).

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**Why do we see the paradox of enrichment?**

It is intuitive that if the carrying capacity of herbivores is higher, then the higher population of herbivores will sustain a higher, more stable population of predators. This is clearly not the case, because the graphs show increasing instability in predator and prey population size as the carrying capacity of the herbivores increases in the graphs above. Because of these fluctuations the predator especially is at a higher risk of extinction due to random fluctuations.

The paradox of enrichment is seen because the predator population is allowed to grow unsustainably large due to the abundance of prey. The predators, who have depleted the prey population, then have a lack of prey. As the predators die, the prey population begins to grow again. However, the lag time can result in the extinction of the predator population.